2.2: Describing Sets

Definition: A set is any collection of objects with no repetitions. An object in a set is said to be an element of the set. One way to write a set is to list them in { } with commas in between the elements.

is an element of

Notation: If A is a set and a is an element of A, we write $a \in A$. If b is not an element of A, we write $b \notin A$.

is not an element of

Example: Write the set of the first five counting numbers and give examples of elements in and not in the set.

$\{1, 2, 3, 4, 5\} = A$

$3 \in A$ $\quad 1 \in A$

$6 \notin A$ $\quad 0 \notin A$ $\quad -3 \notin A$

$\sqrt{17} \notin A$ $\quad \pi \notin A$

Definition: (Set builder notation) Let S be a set. Then we can write $S = \{x \mid x \text{ satisfies some conditions}\}$. This is read "$S$ equals the set of elements x such that x satisfies some conditions".

such that

Another way to think of set builder notation is {form of elements | conditions}. This will show up more in the examples.

Example: Write $S = \{1, 2, 3, 4, 5\}$ in set builder notation.

$\{x \mid x \text{ is one of the first 5 counting numbers}\}$

Definition (Special Sets):

(1) The Natural Numbers: $\mathbb{N} = \{1, 2, 3, 4, ...\}$ $\mathbb{N}$

(2) The Integers: $\mathbb{Z} = \{..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$ $\mathbb{Z}$

(3) The Real Numbers: $\mathbb{R} = \{x \mid x \text{ is any number that can be written as a decimal}\}$

$\mathbb{R}$

Example: Describe the elements of the following sets.

(a) $\{3x \mid x \in \mathbb{Z}\}$ all multiples of 3

$\{\ldots, -9, -6, -3, 0, 3, 6, 9, \ldots\}$

(b) $\{-x \mid x \in \mathbb{N}\}$

$\{-1, -2, -3, -4, \ldots\}$

(c) $\{a/b \mid a, b \in \mathbb{Z}, b \neq 0\}$

all fractions (positive and negative)

(d) $\{x^2 \mid x \in \mathbb{N}\}$

$\{1^2, 2^2, 3^2, 4^2, \ldots\} = \{1, 4, 9, 16, \ldots\}$

Definition: Two sets are equal if they contain exactly the same elements in any order.

Definition: The cardinal number of a set S, denoted $n(S)$ or $|S|$, is the number of elements of S.

Definition: The empty set, denoted $\varnothing$, is the set with no elements. The empty set can also be written as $\{\ \}$.

Definition: A set is a finite set if the cardinal number of the set is 0 or a natural number. A set with infinitely many elements, such as the natural numbers, is called an infinite set.

Example: Find the cardinal number of $A = \{1, 2, 3, 4\}$, $B = \{0\}$, $C = \{2, 4, 6, 8, \ldots\}$, and $\varnothing$.

$n(A) = 4$ $\quad n(B) = 1$ $\quad n(C) = \infty$

$n(\varnothing) = 0$ $\quad$ C is infinite

Example: Find the cardinal number of the following sets.

(a) $S = \{1, 4, 7, 10, 13, ..., 40\}$

d = common difference

$= \frac{40-1}{3} + 1$

$n = \frac{\text{last \#} - \text{first \#}}{d} + 1$

$n(S) = \boxed{14}$

(b) $T = \{33, 37, 41, 45, 49, ..., 353\}$

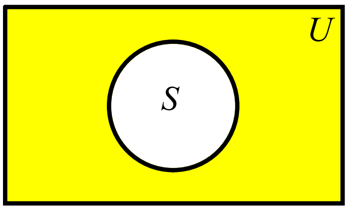
$\frac{353-33}{4} + 1 = \frac{320}{4} + 1 = 81$

$n(T) = \boxed{81}$

Definition: The <u>universal set</u>, denoted U, is the set of all elements being considered in a given discussion.

Definition: The <u>complement</u> of a set S, denoted $\overline{S}$, is the set of all elements in U that are not in S. That is, $\overline{S} = \{x \mid x \in U \text{ and } x \notin S\}$.

A complement can be thought of in the following manner. The shaded region is $\overline{S}$:



Example: If $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, find the complements of $A = \{2, 4, 6, 8\}$ and $B = \{1, 2, 4, 6, 7\}$.

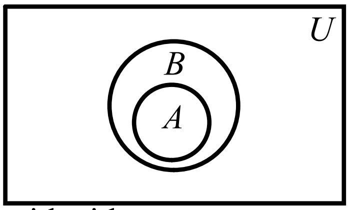
$\overline{A} = \{1, 7, 3, 5\}$

$\overline{B} = \{3, 5, 8\}$

2.2 Completed Notes

Definition: If A and B are sets, we say that A is a <u>subset</u> of B, denoted $A \subseteq B$, if every element of A is an element of B. If $A \subseteq B$ and $A \neq B$, we say that A is a <u>proper subset</u> of B, denoted $A \subset B$.

A subset can be thought of in the following manner. In the figure $A \subseteq B$:



Example: Fill in the blanks with either $\subseteq$ or $\not\subseteq$.

$\{1, 2, 3, 4\}\ \underline{\overset{\subset}{\subseteq}}\ \{1, 2, 3, 4, 5\}$

$\{1, 2, 3, 4\}\ \underline{\overset{=}{\subseteq}}\ \{1, 2, 3, 4\}$

$\{1, 2, 3, 4, 5\}\ \underline{\not\subseteq}\ \{1, 2, 3, 4\}$

$\{1, 2, 3, 4, 5\}\ \underline{\not\subseteq}\ \{1, 2, 3, 4, 6\}$

$\{0\}\ \underline{\not\subseteq}\ \{1, 2, 3, 4\}$

$\varnothing\ \underline{\overset{\subset}{\subseteq}}\ \{1, 2, 3, 4\}$

$\{1, 2, 3, 4\}\ \underline{\not\subseteq}\ \varnothing$

$\varnothing\ \underline{\overset{=}{\subseteq}}\ \varnothing$

Example: Fill in the blanks with either $\in$, $\notin$, $\subseteq$, or $\not\subseteq$.

$\{2\}\ \underline{\overset{\subset}{\subseteq}}\ \{1, 2, 3\}$

$2\ \underline{\in}\ \{1, 2, 3\}$

$5\ \underline{\notin}\ \{1, 2, 3, 4\}$

$\varnothing\ \underline{\overset{\subset}{\subseteq}}\ \{1\}$

$0\ \underline{\notin}\ \varnothing$

$\{4\}\ \underline{\not\subseteq}\ \{2\}$

$0\ \underline{\notin}\ \mathbb{N}$

$\mathbb{Z}\ \underline{\not\subseteq}\ \mathbb{N}$ ← $\mathbb{Z}$ contains 0 and negatives that are not in $\mathbb{N}$

$5\ \underline{\notin}\ \{2x \mid x \in \mathbb{Z}\}$ ← all even integers

$\mathbb{R}\ \underline{\overset{=}{\subseteq}}\ \mathbb{R}$

$\{a/b \mid a, b \in \mathbb{Z}, b \neq 0\}\ \underline{\overset{\subset}{\subseteq}}\ \mathbb{R}$ ← fractions (+ or -)

$\{1.5\}\ \underline{\not\subseteq}\ \mathbb{N}$

First item: if it is a set, use $\subseteq$ or $\not\subseteq$
if it is a number, use $\in$ or $\notin$